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COMPLEX MODEL OF THE EFFICIENCY OF RECTIFICATION PLATES.
4. CROSS MOTION OF PHASES

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A model of mass transfer in the cross motion of phases is proposed in which the compositions of the flows on ideal and real plates at a certain distance h for a vapor and h_1 for a liquid from the site of their injection are equal. The composition of the vapor after a plate is equal to the average value of the compositions of the vapor after the initial and final portions in the direction of liquid motion that, for an ideal plate, are in equilibrium with, respectively, the arriving and discharging liquid. Dependences of the efficiency in the vapor and liquid phases on the parameters of a real plate that are identical and confirm the equality of these efficiencies are derived. An analysis of particular cases of the model for $h = h_1$ and $h = h_1 = 0.5$ and for their boundary values when they are equal to zero or to unity is made. Corresponding relations of the efficiencies are obtained.

Cross motion of phases on a rectification plate is widely used in industry. On one side of a horizontally arranged plate, a liquid flow comes, is depleted by a highly volatile component in motion over it, and is removed on its other side (Fig. 1). The vapor comes from below, is enriched with a highly volatile component in passage through the layer of a liquid on the plate, and is removed from it, arriving at the above-lying plate. The concentration of the highly volatile component in the vapor phase that is removed from a portion of the plate initial in the direction of liquid motion noticeably exceeds the corresponding quantity in the vapor that is removed from the final portion. The vapor escaping from different portions of the plate is partially or totally mixed and its composition is accordingly averaged. Allowance for the equilibrium of the vapor with a liquid arriving at an ideal plate overstates the composition of the averaged vapor, while allowance for the equilibrium with the liquid discharging from the plate understates it.

In the Murphree [1] and Hausen [2] models that have gained the widest acceptance in analysis of the efficiency of rectification columns, the vapor and the liquid leaving an ideal plate are in equilibrium. This condition occurs in the concurrent motion of phases or in the total mixing of a liquid on a plate. Satisfactory agreement of these models with calculated quantities has been obtained in analyzing experimental data derived on laboratory setups or small full-scale plants and also on plates with intense mixing of the liquid [3–6]. However, inspection of rectification columns of larger diameter in which the path length of the liquid on a plate is much larger than the height of the liquid above it shows considerable discrepancies between experimental and calculated data, especially in rectification of multicomponent mixtures [7].

In the model of [8–11], consideration has been given to the equilibrium of a vapor leaving a plate with a liquid at the center of the plate. Because of the difficulty of measuring a given concentration of the liquid in practice, Onda et al. proposed that the concentration be replaced by its average value between the initial and final concentrations.

In uniform depletion of a liquid in the process of its motion on an ideal plate, we can allow an average composition of the escaping vapor equilibrium with the arithmetic-mean composition of the liquid on it, i.e.,

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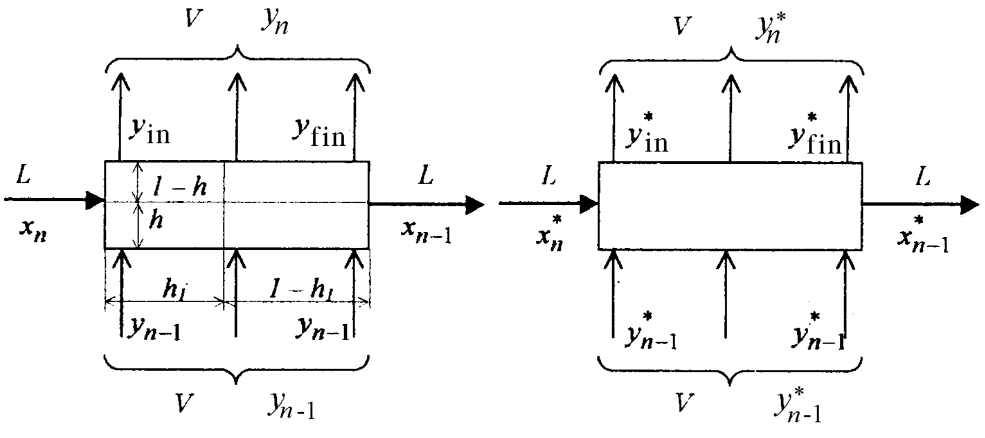

a *b*

Fig. 1. Scheme of the cross motion of phases on real (a) and ideal (b) plates.

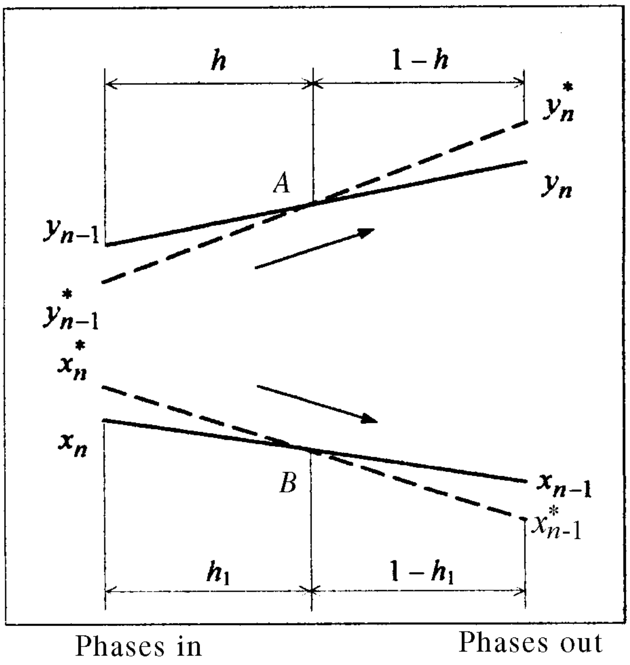

Fig. 2. Change in the concentrations on real (solid lines) and ideal (dashed lines) plates in the cross motion of the phases.

$$y_n^* = \frac{\overset{*}{y}_{\text{in}} + \overset{*}{y}_{\text{fin}}}{2} = \frac{m\,(\overset{*}{x}_{\text{in}} + \overset{*}{x}_{\text{fin}})}{2}\,.$$

In [12, 13], we pointed to certain drawbacks of the known models that are caused by the conditions of the relationship of ideal and real plates. Similar drawbacks also occur in the case of countercurrent motion of the interacting phases where the actual plate is somewhat idealized and its efficiency is overstated.

By analogy with concurrent motion [12], in cross motion of the phases we can use the following model of mass transfer (Fig. 2). The incoming and outgoing flows of a vapor and a liquid on ideal and real plates differ in composition and level off at a certain distance h for the vapor and h_1 for the liquid from the site of their injection (points *A* and *B* in Fig. 2 respectively).

At point A, we express the compositions of the vapor phase in ideal and real plates by the corresponding dependences

$$y_A^* = y_{n-1}^* + h\,(y_n^* - y_{n-1}^*)\,,\quad y_A = y_{n-1} + h\,(y_n - y_{n-1})\,,$$

upon equating which we obtain

$$hy_n^* + (1-h)\,y_{n-1}^* = hy_n + (1-h)\,y_{n-1}\,. \tag{1}$$

For point B we can also obtain a relation similar to (1):

$$(1-h_1)\,x_n^* + h_1 x_{n-1}^* = (1-h_1)\,x_n + h_1\,x_{n-1}\,. \tag{2}$$

In the general case, $h \neq h_1$ but there can be a particular case where the compositions of the vapor and the liquid level off at the same distance in the ideal and real plates.

The equations of material balance of a highly volatile component for ideal and real plates and expressions of the efficiency in the vapor and liquid phases have the same form as in concurrent motion [12]. As has been mentioned above, in cross motion of the phases the composition of the vapor leaving an ideal plate is in equilibrium with a liquid arriving at it and leaving it:

$$y_n^* = m\,\frac{x_n^* + x_{n-1}^*}{2}\,. \tag{3}$$

Simultaneous solution of (1) and of the expression of the efficiency in the vapor phase [12] leads to the dependences

$$y_{n-1}^* = hy_n + (1-h)\,y_{n-1} - h\,\frac{y_n - y_{n-1}}{E_\mathrm{v}}\,; \tag{4}$$

$$y_n^* = hy_n + (1-h)\,y_{n-1} + (1-h)\,\frac{y_n - y_{n-1}}{E_\mathrm{v}}\,, \tag{5}$$

and (2) and (3) with account for (5) lead to

$$x_n^* - x_{n-1}^* = 2\,\frac{1-h_1}{1-2h_1}\,x_n + \frac{2h_1}{1-2h_1}\,x_{n-1} - \frac{2h}{1-2h_1}\,\frac{y_n}{m} - 2\,\frac{1-h}{1-2h_1}\,\frac{y_{n-1}}{m} - 2\,\frac{1-h}{1-2h_1}\,\frac{y_n - y_{n-1}}{mE_\mathrm{v}}\,. \tag{6}$$

By substitution of (4)–(6) into the equation of material balance of an ideal plate with account for the equation of material balance of a real plate [12], we obtain

$$x_n - x_{n-1} = \left(x_{n-1} - \frac{y_{n-1}}{m}\right)E_\mathrm{v} \Big/ \left[L\,(1-E_\mathrm{v})\left(\frac{1-h}{mV} + \frac{1-h_1}{L}\right) + \frac{L}{mV}E_\mathrm{v} - \frac{1}{2}\right]. \tag{7}$$

A similar dependence can be found when the efficiency of mass transfer in a liquid is used. At first, from (2) and the expression of the efficiency in a liquid we derive the dependences

$$x^{*}_{n-1} = (1 - h_1)\, x_n + h_1 x_{n-1} - (1 - h_1)\, \frac{x_n - x_{n-1}}{E_{\text{liq}}}\,; \tag{8}$$

$$x^{*}_{n} = (1 - h_1)\, x_n + h_1 x_{n-1} + h_1\, \frac{x_n - x_{n-1}}{E_{\text{liq}}}\,; \tag{9}$$

then from (1) with account for (3) we derive the difference

$$y^{*}_{n} - y^{*}_{n-1} = \frac{1 - h_1}{1 - h}\, m x_n + \frac{h_1}{1 - h}\, m x_{n-1} - \frac{h}{1 - h}\, y_n - y_{n-1} - \frac{\frac{1}{2} - h_1}{1 - h}\, m\, \frac{x_n - x_{n-1}}{E_{\text{liq}}}\,. \tag{10}$$

In substitution of (8)-(10) into the equation of material balance of an ideal plate with account for the equation of material balance of an actual plate, we obtain

$$x_n - x_{n-1} = \left(x_{n-1} - \frac{y_{n-1}}{m}\right) E_{\text{liq}} \Big/ \left[L\, (1 - E_{\text{liq}}) \left(\frac{1 - h}{mV} + \frac{1 - h_1}{L}\right) + \frac{L}{mV} E_{\text{liq}} - \frac{1}{2}\right].$$

A comparison of the obtained expression with (7) confirms the equality of efficiencies in the vapor and liquid phases for the considered model of mass transfer, i.e.,

$$E_{\text{v}} = E_{\text{liq}} = E_{\text{k}}\,. \tag{11}$$

In the particular case where h can be equal to h_1, expression (7) is transformed to the form

$$x_n - x_{n-1} = \left(x_{n-1} - \frac{y_{n-1}}{m}\right) E_{\text{k}} \Big/ \left[(1 - E_{\text{k}})\, (1 - h) \left(\frac{L}{mV} + 1\right) + \frac{L}{mV} E_{\text{k}} - \frac{1}{2}\right], \tag{12}$$

and for $h = h_1 = 0.5$

$$x_n - x_{n-1} = \left(x_{n-1} - \frac{y_{n-1}}{m}\right) 2E_{\text{k,m}} \Big/ \left(\frac{L}{mV} + \frac{L}{mV} E_{\text{k,m}} - E_{\text{k,m}}\right). \tag{13}$$

Boundary conditions of a generalized model are the conditions of the relationship of ideal and real plates that are inherent in the models of Murphree [1, 4, 5] and Hausen [2, 4, 5] where the distances h and h_1 become equal to zero or to unity. Formula (7), in this case, acquires the corresponding form:

for $y^{*}_{n-1} = y_{n-1}$ and $x^{*}_{n-1} = x_{n-1}$ ($h = 0$, $h_1 = 1$)

$$x_n - x_{n-1} = \left(x_{n-1} - \frac{y_{n-1}}{m}\right) E_{\text{k1}} \Big/ \left(\frac{L}{mV} - \frac{1}{2}\right); \tag{14}$$

for $y^{*}_{n} = y_n$ and $x^{*}_{n} = x_n$ ($h = 1$, $h_1 = 0$)

$$x_n - x_{n-1} = \left(x_{n-1} - \frac{y_{n-1}}{m}\right) E_{\text{k2}} \Big/ \left(\frac{L}{mV} E_{\text{k2}} - E_{\text{k2}} + \frac{1}{2}\right); \tag{15}$$

for $y^{*}_{n-1} = y_{n-1}$ and $x^{*}_{n} = x_n$ ($h = 0$, $h_1 = 0$)

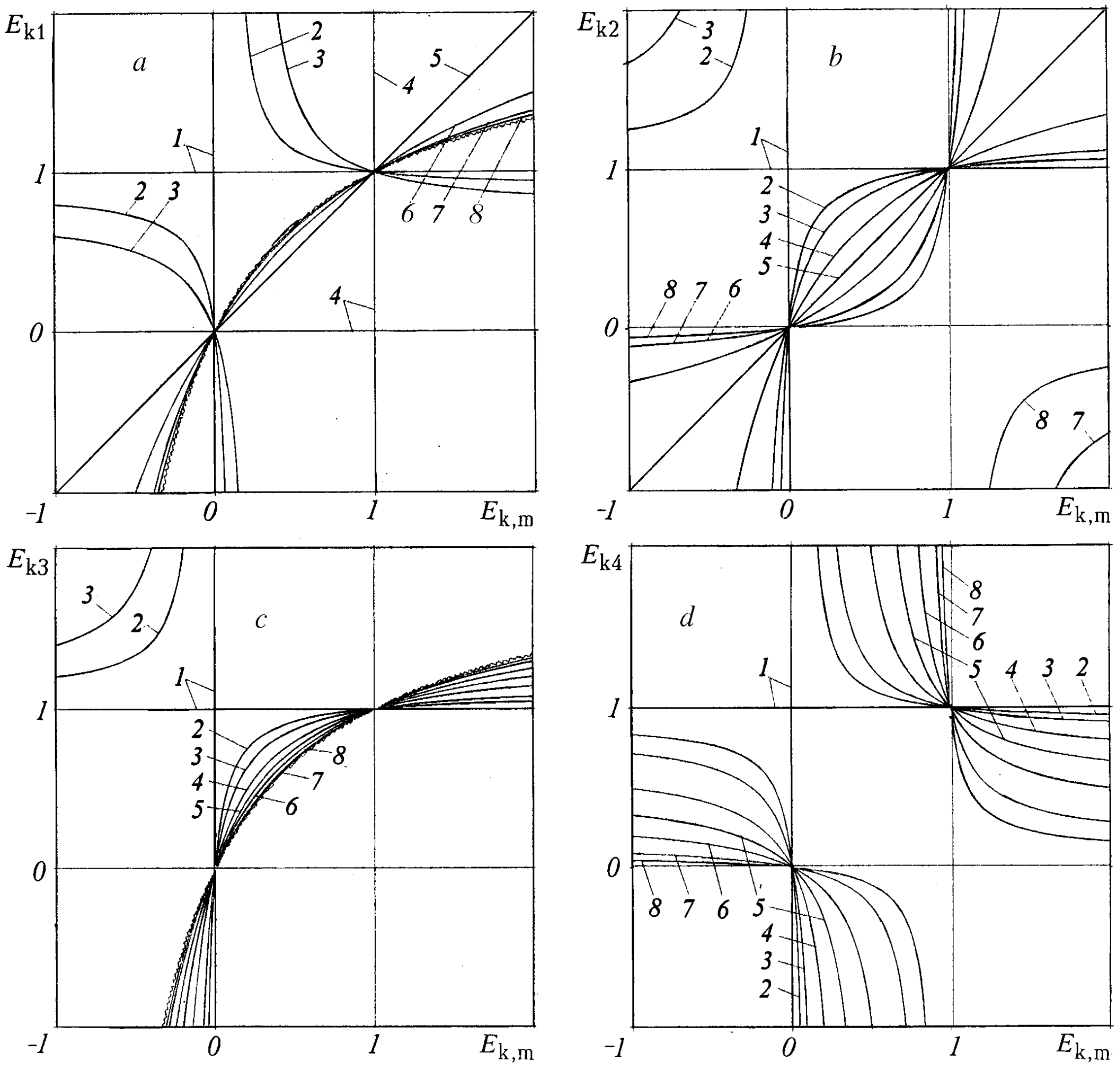

Fig. 3. Dependence of E_{k1} (a), E_{k2} (b), E_{k3} (c), and E_{k4} (d) on $E_{k,m}$ for different values of L/mV: a and b): 1) 0; 2) 0.2; 3) 0.2; 4) 0.5; 5) 1.0; 6) 2.0: 7) 5.0; 8) 10; c and d) 1) 0; 2) 0.1; 3) 0.2; 4) 0.5; 5) 1.0; 6) 2.0; 7) 5.0; 8) 10.

$$x_n - x_{n-1} = \left(x_{n-1} - \frac{y_{n-1}}{m}\right) E_{k3} \Big/ \left(\frac{L}{mV} - E_{k3} + \frac{1}{2}\right); \tag{16}$$

for $y_n^* = y_n$, $x_{n-1}^* = x_{n-1}$ ($h = 1$, $h_1 = 1$)

$$x_n - x_{n-1} = \left(x_{n-1} - \frac{y_{n-1}}{m}\right) E_{k4} \Big/ \left(\frac{L}{mV} E_{k4} - \frac{1}{2}\right). \tag{17}$$

The left-hand sides in formulas (13)–(17) are equal. By equating their right-hand sides we obtain

$$\frac{\frac{L}{mV}}{2E_{k,m}} + \frac{L}{2mV} - \frac{1}{2} = \frac{\frac{L}{mV} - \frac{1}{2}}{E_{k1}} = \frac{L}{mV} - 1 + \frac{1}{2E_{k2}} = \frac{\frac{L}{mV} + \frac{1}{2}}{E_{k3}} - 1 = \frac{L}{mV} - \frac{1}{2E_{k4}}. \tag{18}$$

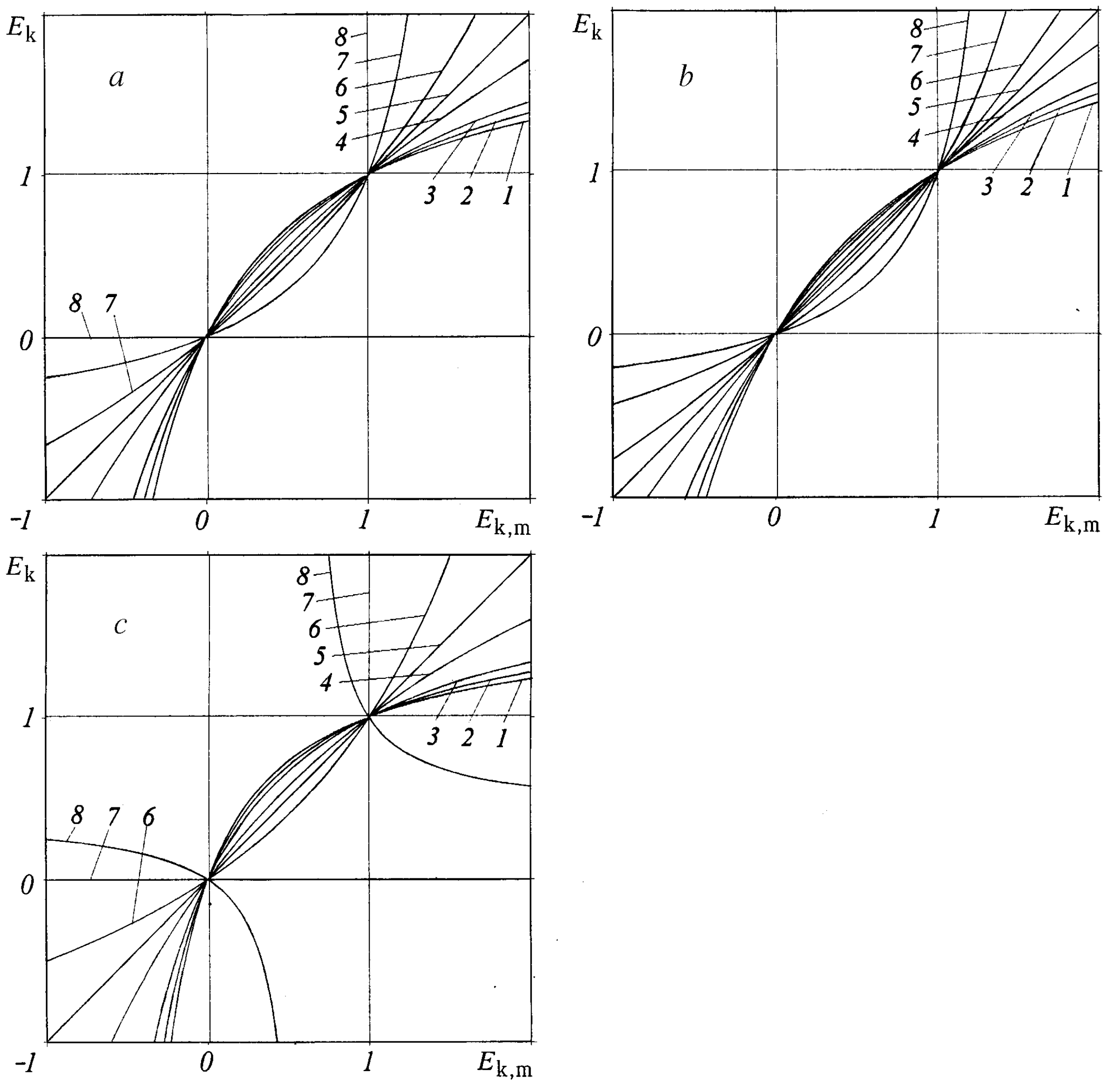

Fig. 4. Dependence of E_k on $E_{k,m}$ for L/mV = 1.5, h_1 = 0.5, and different values of h_1 (b) and also for different values of $h_1 = h$ (c): 1) 0; 2) 0.1; 3) 0.2; 4) 0.4; 5) 0.5; 6) 0.6; 7) 0.8; 8) 1.0.

The efficiency of a real plate with the known experimental data can be calculated from formulas derived from (7) and (13), respectively:

$$E_k = \left[(1-h)\frac{L}{mV} - h_1 + \frac{1}{2}\right] \Bigg/ \left(\frac{x_n - \dfrac{y_{n-1}}{m}}{x_n - x_{n-1}} - h\frac{L}{mV} - h_1 \right); \tag{19}$$

$$E_{k,m} = \frac{L}{mV} \Bigg/ \left(\frac{x_n + x_{n-1} - 2\dfrac{y_{n-1}}{m}}{x_n - x_{n-1}} - \frac{L}{mV} \right). \tag{20}$$

Figure 3 graphically shows the relation of the pairs of efficiencies of actual plates. The efficiency E_{k1} that reflects the conditions of the relationship of ideal and real plates in the Murphree model in analyzing the efficiency in the vapor phase and the efficiency $E_{k,m}$ have real values in the range $0.5 < L/mV < 10$ (Fig. 3a). The quantity E_{k1} is higher than $E_{k,m}$ for $L/mV < 1$, equal to it for $L/mV = 1$, and lower than it for $L/mV > 1$. When it is required to reduce the efficiency in calculational procedures, use of $E_{k,m}$ is more preferable. For values of $L/mV < 0.5$, the interrelationship of the indicated efficiencies is possible just for the values of one efficiency being in the unreal range where they are either less than zero or more than unity. Consequently, this region is of no practical interest.

With change in L/mV the dependence of the efficiency E_{k2} that reflects the conditions of the relationship of ideal and real plates in the Murphree model in analyzing the efficiency in a liquid on $E_{k,m}$ is possible in a wider range of L/mV (Fig. 3b). The efficiency E_{k2} is higher than $E_{k,m}$ for $L/mV < 1$ and lower than $E_{k,m}$ for $L/mV > 1$. Therefore, if there is a need to reduce the efficiency in the calculations it is appropriate to use $E_{k,m}$ in the first case and E_{k2} in the second case.

The change in L/mV has a smaller effect on the dependence of the efficiency E_{k3} that takes into account the conditions of the relationship of ideal and real plates in the Hausen model on $E_{k,m}$ (Fig. 3c). Just as in the previous case, the real values of one efficiency correspond to the real values of the other. For any values of L/mV, the efficiency E_{k3} is always higher than $E_{k,m}$. In this connection, if there is a need to reduce the calculated value of the efficiency, it is appropriate to use $E_{k,m}$ that is inherent in the complex model of mass transfer.

As is obvious from Fig. 3d, for distances h and h_1 equal to unity real E_{k4} correspond to unreal $E_{k,m}$ and vice versa for any values of L/mV. Therefore, it seems impossible to use E_{k4} and formulas that involve it in calculations of the real parameters of a plate.

The dependence of the efficiencies on h and h_1 for $L/mV = 1.5$ are presented in Fig. 4. For a fixed value of one of these distances equal to half of the total value, a decrease in the other improves the efficiency (Fig. 4a and b). For $h_1 = 0.5$ and $h = 1$ (Fig. 4a), the efficiency E_k is equal to either zero or unity. Therefore use of the calculational dependences with the indicated values of the distances loses its meaning. Simultaneous change in h and h_1 (Fig. 4c) has the same character as with a change in one of them – an increase in the distances reduces the efficiency. The real values of E_k and formulas involving it are possible for $h = h_1 < 0.8$. This fact explains the absence of the real values of the efficiencies presented in Fig. 3d for which $h = h_1 = 1$.

NOTATION

E, efficiency of the plate; h and h_1, distance from the site of injection of the vapor and the liquid, respectively, to the surface of equality of the concentrations of the phases in the ideal and actual plates; L, molar flow of the liquid; m, coefficient of equilibrium; V, molar flow of the vapor; x and y, concentration of the highly volatile component in the liquid and the vapor, respectively. Subscripts: k, cross motion of the phases; k,m, cross motion of the phases for values of h and h_1 equal to half of their total value; fin, final portion; liq, liquid phase; in, initial portion; n, number of the considered plate; $n - 1$, number of the preceding plate in the direction of vapor motion; v, vapor phase. Superscript: *, ideal conditions.

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